OCR

Oxford Cambridge and RSA

Tuesday 16 June 2015 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Duration: 1 hour 30 minutes

309302703030

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

* The Question Paper will be found inside the Printed Answer Book.
* Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
* **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
* Use black ink. HB pencil may be used for graphs and diagrams only.
* Read each question carefully. Make sure you know what you have to do before starting your answer.
* Answer **all** the questions.
* Do **not** write in the bar codes.
* You are permitted to use a scientific or graphical calculator in this paper.
* Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

* The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
* You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
* The total number of marks for this paper is **72**.
* The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
* This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

* Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Turn over

Section A (36 marks)

1 Solve the equation $\dfrac{5x}{2x+1} - \dfrac{3}{x+1} = 1$. **[5]**

2 Express $6\cos 2\theta + \sin\theta$ in terms of $\sin\theta$.

Hence solve the equation $6\cos 2\theta + \sin\theta = 0$, for $0° \leqslant \theta \leqslant 360°$. **[7]**

3 **(i)** Find the first three terms of the binomial expansion of $\dfrac{1}{\sqrt[3]{1-2x}}$. State the set of values of x for which the expansion is valid. **[5]**

 (ii) Hence find a and b such that $\dfrac{1-3x}{\sqrt[3]{1-2x}} = 1 + ax + bx^2 + \dots$. **[3]**

4 You are given that $f(x) = \cos x + \lambda \sin x$ where λ is a positive constant.

 (i) Express $f(x)$ in the form $R\cos(x-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving R and α in terms of λ. **[4]**

 (ii) Given that the maximum value (as x varies) of $f(x)$ is 2, find R, λ and α, giving your answers in exact form. **[4]**

5 A curve has parametric equations $x = \sec\theta$, $y = 2\tan\theta$.

 (i) Given that the derivative of $\sec\theta$ is $\sec\theta\tan\theta$, show that $\dfrac{dy}{dx} = 2\operatorname{cosec}\theta$. **[3]**

 (ii) Verify that the cartesian equation of the curve is $y^2 = 4x^2 - 4$. **[2]**

Fig. 5 shows the region enclosed by the curve and the line $x = 2$. This region is rotated through 180° about the x-axis.
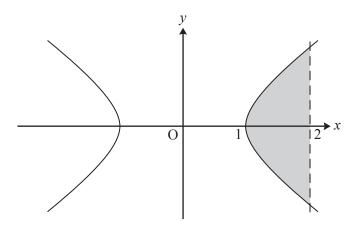

Fig. 5

 (iii) Find the volume of revolution produced, giving your answer in exact form. **[3]**

Section B (36 marks)

6 Fig. 6 shows a lean-to greenhouse ABCDHEFG. With respect to coordinate axes O*xyz*, the coordinates of the vertices are as shown. All distances are in metres. Ground level is the plane $z = 0$.
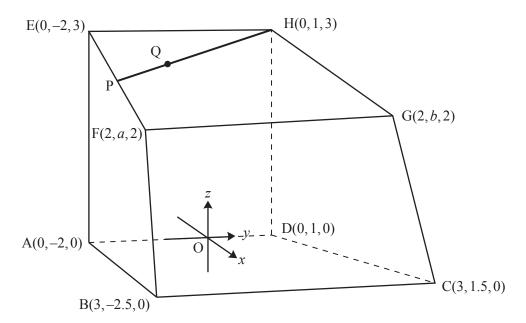

Fig. 6

(i) Verify that the equation of the plane through A, B and E is $x + 6y + 12 = 0$.

Hence, given that F lies in this plane, show that $a = -2\frac{1}{3}$. **[4]**

(ii) (*A*) Show that the vector $\begin{pmatrix} 1 \\ -6 \\ 0 \end{pmatrix}$ is normal to the plane DHC. **[2]**

(*B*) Hence find the cartesian equation of this plane. **[2]**

(*C*) Given that G lies in the plane DHC, find *b* and the length FG. **[2]**

(iii) Find the angle EFB. **[5]**

A straight wire joins point H to a point P which is half way between E and F. Q is a point two-thirds of the way along this wire, so that HQ = 2QP.

(iv) Find the height of Q above the ground. **[3]**

Question 7 begins on page 4.

7 A drug is administered by an intravenous drip. The concentration, x, of the drug in the blood is measured as a fraction of its maximum level. The drug concentration after t hours is modelled by the differential equation

$$\frac{dx}{dt} = k(1 + x - 2x^2),$$

where $0 \leqslant x < 1$, and k is a positive constant. Initially, $x = 0$.

(i) Express $\dfrac{1}{(1+2x)(1-x)}$ in partial fractions. **[3]**

(ii) Hence solve the differential equation to show that $\dfrac{1+2x}{1-x} = e^{3kt}$. **[7]**

(iii) After 1 hour the drug concentration reaches 75% of its maximum value and so $x = 0.75$.

Find the value of k, and the time taken for the drug concentration to reach 90% of its maximum value. **[3]**

(iv) Rearrange the equation in part **(ii)** to show that $x = \dfrac{1 - e^{-3kt}}{1 + 2e^{-3kt}}$.

Verify that in the long term the drug concentration approaches its maximum value. **[5]**

END OF QUESTION PAPER

OCR

Oxford Cambridge and RSA

Tuesday 16 June 2015 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01   Applications of Advanced Mathematics (C4)

INSTRUCTIONS

The examination is in two parts:

> Paper A (1 hour 30 minutes)
> Paper B (up to 1 hour)

Supervisors are requested to ensure that Paper B **is not issued** until Paper A has been collected in from the candidates.

Centres may, if they wish, grant a supervised break between the two parts of this examination.

Paper B should not be attached to the corresponding paper A script. For Paper A only the candidates' printed answer books, in the same order as the attendance register, should be sent for marking; the question paper should be retained in the centre or recycled. For Paper B only the question papers, on which the candidates have written their answers, should be sent for marking; the insert should be retained in the centre or recycled. Any additional sheets used must be carefully attached to the correct paper.

For Paper B (Comprehension) only.

A standard English dictionary is allowed for the comprehension.
(Dictionaries to be used in the examination must be thoroughly checked before the examination.) Full regulations are in the JCQ Regulations and Guidance booklet.

This notice must be on the invigilator's desk at all times during the morning of Tuesday 16 June 2015.

OCR
Oxford Cambridge and RSA

Tuesday 16 June 2015 – Afternoon

A2 GCE MATHEMATICS MEI

4754/01A Applications of Advanced Mathematics (C4) Paper A

PRINTED ANSWER BOOK

Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number					

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
* The Question Paper will be found inside the Printed Answer Book.
* Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
* **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
* Use black ink. HB pencil may be used for graphs and diagrams only.
* Read each question carefully. Make sure you know what you have to do before starting your answer.
* Answer **all** the questions.
* Do **not** write in the bar codes.
* You are permitted to use a scientific or graphical calculator in this paper.
* Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
* The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
* You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
* The total number of marks for this paper is **72**.
* The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
* This paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

1

2

Turn over

3 (i)

3 (ii)

4 (i)

4 (ii)

5(i)

5(ii)

Turn over

5 (iii)

Section B (36 marks)

6 (i)

Turn over

6 (ii)(*A*)

6 (ii)(*B*)

6 (ii)(*C*)

Turn over

6 (iii)

6 (iv)

Turn over

7 (i)

7 (ii)

(answer space continued on next page)

7 (ii) (continued)

7 (iii)

7 (ii) (continued)

7 (iv)

1. **Annotations and abbreviations**

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. **Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand**

   a   Annotations should be used whenever appropriate during your marking.

        The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

        For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

   b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

        Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

   c   The following types of marks are available.

        M
        A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

        A
        Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

        B
        Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g   Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question	Answer	Marks	Guidance
1	$\Rightarrow 5x(x+1) - 3(2x+1) = (2x+1)(x+1)$	M1*	Multiplying throughout by $(2x+1)(x+1)$ or combining fractions and multiplying up oe (eg can retain denominator throughout)
			Condone a single numerical error, sign error or slip provided that there is no conceptual error in the process involved
			Do not condone omission of brackets unless it is clear from subsequent work that they were assumed
			eg $5x(x+1) - 3(2x+1) = (2x+1)(x \underline{-1})$ gets M1
			$5x(x+1) - 3(2x+1) = 1$ gets M0
			$5x(x+1)(2x+1) - 3(2x+1)(x+1) = (x+1)(2x+1)$ gets M0
			$5x(x+1) - 3(2x+1) = (2x+1)$ gets M1, just, for slip in omission of $(x+1)$
		M1dep*	Multiplying out, collecting like terms and forming quadratic (= 0). Follow through from their equation provided the algebra is not significantly eased and it is a quadratic. Condone a further sign or numerical error or a minor slip when rearranging
	$\Rightarrow \quad 3x^2 - 4x - 4 = 0$	A1	oe www (not fortuitously obtained – check for double errors)
	$\Rightarrow \quad (3x+2)(x-2) = 0$	M1	Solving their three term quadratic (= 0) provided $b^2 - 4ac \geq 0$. Use of correct quadratic equation formula (if formula is quoted correctly then only one sign slip is permitted, if the formula is quoted incorrectly M0, if not quoted at all substitution must be completely correct to earn the M1) or factorising (giving their x^2 term and one other term when factors multiplied out) or comp. the square (must get to the square root stage involving $\pm$ and arithmetical errors may be condoned provided their $3(x - 2/3)^2$ seen or implied)
	$\Rightarrow \quad x = -2/3$ or 2	A1	cao for both obtained www (condone -0.667 or better) (If no factorisation (oe) seen B1 for each answer stated following correct quadratic)
		[5]	

Question	Answer	Marks	Guidance
2	$\cos 2\theta = 1 - 2\sin^2\theta$	M1*	$\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ (maybe implied in substitution)
	$(6\cos 2\theta + \sin\theta =)\; 6 - 12\sin^2\theta + \sin\theta$	A1	
	$6\cos 2\theta + \sin\theta = 0$ $\Rightarrow \quad 12\sin^2\theta - \sin\theta - 6 = 0$ $\Rightarrow \quad (4\sin\theta - 3)(3\sin\theta + 2) = 0$	M1dep*	Use of correct quadratic equation formula or factorising or comp. the square on their three term quadratic in $\sin\theta$ (see guidance in question 1 for awarding this method mark) provided $b^2 - 4ac \geq 0$
	$\Rightarrow \quad \sin\theta = 3/4 \text{ or } -2/3$	A1	www
		B1	First correct solution to 1 dp or better (eg 48.59° etc)
		B1	Three correct solutions
	$\Rightarrow \quad \sin\theta = 3/4,\ \theta = 48.6°,\ 131.4°$ $\sin\theta = -2/3,\ \theta = 221.8°,\ 318.2°$	B1	All four correct solutions and no others in the range
			Ignore solutions outside the range
			SC Award max B1B1B0 for answers in radians (0.85, 2.29, 3.87, 5.55 or better – so one correct B1, three correct B1). Award max B1 if there are extra solutions in the range with radians
			SC If M1M1 awarded and both values of $\left|\sin\theta\right| \leq 1$ but B0B0B0 then award B1 only for evidence of using $\sin\theta \equiv \sin(180 - \theta)$
		[7]	

Question	Answer	Marks	Guidance
3 (i)	$$\frac{1}{\sqrt[3]{1-2x}} = (1-2x)^{-1/3}$$	B1	$n = -1/3$. See below **SC** for those with $n = 1/3$
	$$= 1 + (-\frac{1}{3})(-2x) + \frac{(-\frac{1}{3})(-\frac{4}{3})}{2!}(-2x)^2 + ...$$	M1	All three **correct unsimplified binomial** coefficients (not nCr) soi condone absence of brackets only if it is clear from subsequent work that they were assumed
		B1	$1 + (2/3)x + ...$ www
	$$= 1 + \frac{2}{3}x + \frac{8}{9}x^2 + ...$$	B1	$(8/9)x^2$ www in this term
			If there is an error, in say, the third coefficient of the expansion then M0B1B0 is possible
			SC For $n = 1/3$ award B1 for $1 - (2/3)x$ and B1 for $-(4/9)x^2$ (so max 2 out of the first 4 marks)
	Valid for $-\frac{1}{2} < x < \frac{1}{2}$ or $|x| < \frac{1}{2}$	B1	Independent of expansion. Accept, say, $-1/2 < |x| < 1/2$ or $-1/2 \le x < 1/2$ (must be strict inequality for $+1/2$)
		[5]	
3 (ii)	$$\frac{1-3x}{\sqrt[3]{1-2x}} = (1-3x)(1 + \frac{2}{3}x + \frac{8}{9}x^2 + ...)$$		
	$$= 1 + \frac{2}{3}x + \frac{8}{9}x^2 - 3x - 2x^2 + ...$$	M1	Use of $(1-3x) \times$ their $\left(1 + (2/3)x + (8/9)x^2 + \cdots\right)$ and attempt at removal of brackets (condone absence of brackets but must have two terms in x and two terms in x^2)
		A1ft	Correct simplified expansion following their expansion in (i). This mark is dependent on scoring both M marks in (i) and (ii)
	$$= 1 - \frac{7}{3}x - \frac{10}{9}x^2 + ...$$	A1	cao or B3 www in either part **SC** following either M0 or M1, B1 for either a or b correct
		[3]	

Question	Answer	Marks	Guidance
4 (i)	$\cos x + \lambda \sin x = R \cos(x - \alpha)$ $\qquad = R \cos x \cos \alpha + R \sin x \sin \alpha$ $\Rightarrow \quad R \cos \alpha = 1, R \sin \alpha = \lambda$	M1	Correct pairs. Condone sign error (so accept $R \sin \alpha = -\lambda$)
	$\Rightarrow \quad R^2 = 1 + \lambda^2, R = \surd(1 + \lambda^2)$	B1	Positive square root only - isw. Accept $R = 1/\cos(\arctan \lambda)$ or $R = \lambda / \sin(\arctan \lambda)$
	$\tan \alpha = \lambda$ (oe)	M1	Follow through their pairs. $\tan \alpha = \lambda$ with no working implies both M marks. However, $\cos \alpha = 1$, $\sin \alpha = \lambda \Rightarrow \tan \alpha = \lambda$ scores M0M1. First two M marks may be implied by combining one of the pairs with R, eg, $\cos \alpha = \dfrac{1}{\surd\left(1 + \lambda^2\right)}$ or $\sin \alpha = \dfrac{\lambda}{\surd(1 + \lambda^2)}$
	$\Rightarrow \alpha = \arctan \lambda$ (oe)	A1	$\alpha = \arccos\left(\dfrac{1}{\surd\left(1 + \lambda^2\right)}\right)$, $\alpha = \arcsin\left(\dfrac{\lambda}{\surd\left(1 + \lambda^2\right)}\right)$ Accept embedded answers, eg, $\surd\left(1 + \lambda^2\right)\cos\left(x - \arctan \lambda\right)$ for full marks
		[4]	
4 (ii)	max is R so $R = 2$	B1	
	$1 + \lambda^2 = 4 \Rightarrow \lambda = \surd 3$	M1 A1	M1 for using their $\surd(1 + \lambda^2) = R_{max}$, A0 for $\pm\surd 3$ as final answer
	$\alpha = \arctan \surd 3 = \pi/3$	B1	www (eg $\lambda = 1$ and $\cos \alpha = (1 + \lambda)^{-1} \Rightarrow \alpha = \pi/3$ is B0) Exact answers only for final A and B marks
		[4]	

Question			Answer	Marks	Guidance
5	(i)		$x = \sec\theta,\ y = 2\tan\theta$		
			$\Rightarrow\quad \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{2\sec^2\theta}{\sec\theta\tan\theta}$	M1A1	M1 for their $(dy/d\theta)\div\sec\theta\tan\theta$ in terms of θ A1 cao (oe) allow for unsimplified form even if subsequently cancelled incorrectly ie can isw
			$= \dfrac{2\sec\theta}{\tan\theta} = \dfrac{2}{\cos\theta}\cdot\dfrac{\cos\theta}{\sin\theta} = \dfrac{2}{\sin\theta} = 2\cosec\theta *$	A1	cao www (NB **AG**) – must be at least one intermediate step between $\dfrac{2\sec\theta}{\tan\theta}$ (oe) and either $\dfrac{2}{\sin\theta}$ or $2\cosec\theta$
				[3]	
5	(ii)		$x^2 = \sec^2\theta = 1 + \tan^2\theta = 1 + \frac{1}{4}y^2$	M1	$\sec^2\theta = 1 + \tan^2\theta$ (oe) used
			$\Rightarrow\quad y^2 = 4(x^2 - 1) = 4x^2 - 4 *$	A1	www NB **AG**
				[2]	
		OR	$4\tan^2\theta = 4\sec^2\theta - 4$	B1*	Correct substitution of x and y into the given answer
			$\Rightarrow 1 + \tan^2\theta = \sec^2\theta$ which is true	B1dep*	Dependent on previous mark – must simplify/remove the factor of 4 from each term and state that the correctly derived trig identity is true
5	(iii)		$V = \pi\displaystyle\int_1^2 y^2\,dx = \pi\int_1^2 (4x^2 - 4)\,dx$	M1	$k\pi\displaystyle\int_1^2 \left(4x^2 - 4\right)(dx)$ with $k = 1$ or $1/2$, allow correct limits later condone lack of dx
			$\dfrac{4}{3}x^3 - 4x$	B1	$(4/3)x^3 - 4x\ \left(\text{or } (2/3)x^3 - 2x\right)$
			$= \pi\left[\dfrac{4}{3}x^3 - 4x\right]_1^2 = \dfrac{16}{3}\pi$	A1	exact – mark **final** answer
				[3]	

	Question		Answer	Marks	Guidance
6	**(i)**		A: $0 + 6.(-2) + 12 = 0$ B: $3 + 6.(-2.5) + 12 = 0$ E: $0 + 6.(-2) + 12 = 0$	B2,1,0	B1 for two points verified (must see as a minimum $-12 + 12 = 0$, $3 - 15 + 12 = 0$, $-12 + 12 = 0$) or any valid complete method for either finding or verifying that $x + 6y + 12 = 0$ gets M1 A1
			At F, $2 + 6a + 12 = 0$	M1	Substitution of F into $x + 6y + 12 = 0$
			$\Rightarrow$ $6a = -14, a = -14/6 = -7/3$ *	A1	www NB **AG**
				[4]	
6	**(ii)**	**(A)**	$\overrightarrow{DH}.\begin{pmatrix}1\\-6\\0\end{pmatrix} = \begin{pmatrix}1\\-6\\0\end{pmatrix}\begin{pmatrix}0\\0\\3\end{pmatrix} = 1\times0 + (-6)\times0 + 0\times3 = 0$	B1	scalar product with a **direction** vector in the plane (including evaluation and $= 0$) (**OR** M1 forms a vector product with at least two correct terms in solution)
			$\overrightarrow{DC}.\begin{pmatrix}1\\-6\\0\end{pmatrix} = \begin{pmatrix}3\\0.5\\0\end{pmatrix}.\begin{pmatrix}1\\-6\\0\end{pmatrix} = 3\times1 + 0.5\times(-6) + 0\times0 = 0$	B1	scalar product with second **direction** vector, with evaluation. (following OR above, A1 correct ie a multiple of $\mathbf{i} - 6\mathbf{j}$)
				[2]	(NB finding only one direction vector and its scalar product is B1 only)
6	**(ii)**	**(B)**	$\mathbf{r}.(\mathbf{i} - 6\mathbf{j}) = \mathbf{j}.(\mathbf{i} - 6\mathbf{j})$	M1	$\mathbf{r}\cdot\mathbf{n} = \mathbf{a}\cdot\mathbf{n}$ with $\mathbf{n} = \begin{pmatrix}1\\-6\\0\end{pmatrix}$ and $\mathbf{a} = \begin{pmatrix}0\\1\\3\end{pmatrix}$ or $\begin{pmatrix}0\\1\\0\end{pmatrix}$ or $\begin{pmatrix}3\\1.5\\0\end{pmatrix}$ **or**
					substituting H(0, 1, 3) or D(0, 1, 0) or C(3, 1.5, 0) into $x - 6y = d$
			$\Rightarrow x - 6y + 6 = 0$	A1	oe (isw if d found correctly and $x - 6y = d$ stated) B2 www correct equation stated
				[2]	
6	**(ii)**	**(C)**	$2 - 6b + 6 = 0 \Rightarrow b = 4/3$	B1	oe – exact answer
			$FG = 1\frac{1}{3} + 2\frac{1}{3} = 3\frac{2}{3}$	B1 **[2]**	oe – exact answer

Question			Answer	Marks	Guidance
6	(iii)		$(\overrightarrow{FE} =) -2\mathbf{i} + (1/3)\mathbf{j} + \mathbf{k}$, $(\overrightarrow{FB} =) \mathbf{i} - (1/6)\mathbf{j} - 2\mathbf{k}$	B1 B1	or $(\overrightarrow{EF} =) 2\mathbf{i} + (-1/3)\mathbf{j} - \mathbf{k}$ or $(\overrightarrow{BF} =) -\mathbf{i} + (1/6)\mathbf{j} + 2\mathbf{k}$
			$\cos\theta = \dfrac{-2(1) + (1/3)(-1/6) + 1(-2)}{\sqrt{4 + 1/9 + 1}\sqrt{1 + 1/36 + 4}}$	M1	$\cos\theta = \left(\overrightarrow{FE} \cdot \overrightarrow{FB}\right) / \left(\left\|\overrightarrow{FE}\right\|\left\|\overrightarrow{FB}\right\|\right)$ (oe) follow through their FE and FB (allow any combination of FE, EF with FB, BF) – allow one sign slip only
			$\theta = \arccos\left(\dfrac{-\dfrac{73}{18}}{\dfrac{\sqrt{46}}{3} \times \dfrac{\sqrt{181}}{6}}\right)$	A1	$\arccos\left(\pm\dfrac{-2 - 1/18 - 2}{5.069}\right) = \arccos\left(\pm -0.800\right)$
			$\Rightarrow \quad q = 143°$	A1	3sf or better (or 2.5(0) radians or better). Allow candidates who find the acute angle using either $\overrightarrow{EF}$ with $\overrightarrow{FB}$ or $\overrightarrow{FE}$ with $\overrightarrow{BF}$ and then state the obtuse angle. **Do not isw those who find the obtuse angle and then state the acute angle.** Note: $90 + 2\arctan(1/2)$ is 0/5
				[5]	
	OR		$EF = \sqrt{46}/3$, $FB = \sqrt{181}/6$, $EB = \sqrt{73}/2$	B3,2,1,0	One mark for each (2.26, 2.24, 4.27)
			$\theta = \arccos\left(\dfrac{\left(\sqrt{46}/3\right)^2 + \left(\sqrt{181}/6\right)^2 - \left(\sqrt{73}/2\right)^2}{2(\sqrt{46}/3)(\sqrt{181}/6)}\right)$	M1 A1	cosine rule correct with their EF, FB, EB $q = 143°$
6	(iv)		z coordinate of P is 5/2	B1	stating the correct z-coordinate of P; ignore incorrect x and y coordinates (or stated in a position vector)
			$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ} = \begin{pmatrix} 1 \\ -13/6 \\ 5/2 \end{pmatrix} + \dfrac{1}{3}\left(\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -13/6 \\ 5/2 \end{pmatrix}\right)$	M1	Complete method for finding the z-coordinate of Q or $\overrightarrow{OQ} = (\overrightarrow{OH}) + (2/3)(\overrightarrow{HP})$ or $\overrightarrow{OQ} = (2/3)(\overrightarrow{OP}) + (1/3)(\overrightarrow{OH})$
			so height of Q is 8/3 (metres above ground)	A1	2.67 or better
				[3]	

Question	Answer	Marks	Guidance
7 (i)	$\dfrac{1}{(1+2x)(1-x)} = \dfrac{A}{1+2x} + \dfrac{B}{1-x} \Rightarrow 1 = A(1-x) + B(1+2x)$		
		M1	Cover up, substitution or equating coefficients
	$x = 1 \Rightarrow 3B = 1,\ B = 1/3$	A1	
	$x = -\frac{1}{2} \Rightarrow 1 = 3A/2\ ,\ A = 2/3$	A1	isw after correct A and B stated
		[3]	
7 (ii)	$1 + x - 2x^2 = (1+2x)(1-x)$	B1	May be seen in separation of variables (may be implied by later working) – implied by the use of factors $(1 + 2x)$ and $(1 - x)$
	$\Rightarrow \dfrac{1}{3}\int\left[\dfrac{2}{(1+2x)} + \dfrac{1}{1-x}\right]dx = \int k\,dt$	M1	Separating variables and substituting partial fractions. If no subsequent work integral signs needed, but allow omission of dx or dt , but must be correctly placed if present
	$\lambda \ln(1+2x) + \mu \ln(1-x) = kt (+c)$	A1	Any non-zero constant λ, μ
	$\Rightarrow \quad \ln(1+2x) - \ln(1-x) = 3kt\ (+c)$	A1	www oe (condone absence of c)
	When $t = 0, x = 0 \Rightarrow c = 0$	B1	cao (**must** follow previous A1) need to **show** (at some stage) that $c = 0$. As a minimum $t = 0, x = 0, c = 0$. Note that $c = \ln(-1)$ (usually from incorrect integration of $(1 - x)$) or similar scores B0
	$\Rightarrow \quad \ln\left(\dfrac{1+2x}{1-x}\right) = 3kt$	M1	Combining both their log terms correctly. Follow through their c. Allow if $c = 0$ clearly stated (provided that $c = 0$) even if B mark is not awarded, but do not allow if c omitted
	$\Rightarrow \quad \dfrac{1+2x}{1-x} = e^{3kt}$ *	A1	**AG** www must have obtained all previous marks in this part
		[7]	

Question			Answer	Marks	Guidance
7	(iii)		$(1 + 2(0.75)) / (1 - 0.75) = e^{3k}$	M1	substituting $t = 1$, $x = 0.75$ at any stage
			$k = (1/3)\ln 10 \; (= 0.768 \; (3 \text{ s.f.}))$	A1	3sf or better
			$t = \ln(2.8/0.1)/3k = 1.45$ hours	A1	1.45 (or better) or 1 hr 27 mins
				[3]	
7	(iv)		$1 + 2x = e^{3kt} - xe^{3kt}$		
			$\Rightarrow \quad 2x + xe^{3kt} = e^{3kt} - 1$	M1*	Multiplying out and collecting x terms (condone one error)
			$\Rightarrow \quad x(2 + e^{3kt}) = e^{3kt} - 1$	M1dep*	Factorising their x terms correctly
			$\Rightarrow \quad x = (e^{3kt} - 1)/(2 + e^{3kt})$	A1	
			$= (1 - e^{-3kt})/(1 + 2e^{-3kt})$ *	A1	www (**AG**) – as AG must be an indication of how previous line leads to the required result (eg stating or showing multiplying by e^{-3kt})
			when $t \to \infty$ $e^{-3kt} \to 0$ $x = (1 - e^{-3kt})/(1 + 2e^{-3kt}) \to 1/1 = 1$	B1	clear indication that $e^{-3kt} \to 0$ so, for example, accept as a minimum $(x \to) \dfrac{1-0}{1+0} = 1$ **or** $e^{-3kt} \to 0 \Rightarrow (x \to) 1$ (NB substitution of large values of t with no further explanation is B0)
				[5]	
		OR	$\dfrac{1-x}{1+2x} = e^{-3kt}$	B1	
			$1 - x = e^{-3kt} + 2xe^{-3kt}$	M1*	Multiplying up and expanding (condone one error)
			$x(1 + 2e^{-3kt}) = 1 - e^{-3kt}$	M1dep*	Factorising their x terms correctly
			$x = (1 - e^{-3kt})/(1 + 2e^{-3kt})$ *	A1	www (**AG**) – final B mark as in scheme above

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance
programme your call may be recorded or monitored

PART OF THE
CAMBRIDGE ASSESSMENT
GROUP

UKAS
MANAGEMENT
SYSTEMS
001

4754 Applications of Advanced Mathematics (C4)

General Comments

Candidates found Paper A more straightforward than in previous years and therefore the standard of work was very high. This paper was accessible to all candidates but there were sufficient questions for the more able candidates to show their skills.

Paper B, the comprehension, was well understood and most candidates scored good marks here.

Candidates made similar errors as in previous years and these included:

- Sign and basic algebraic errors (Question 1)
- Failure to include a constant of integration (Question 7(ii))
- Poor anti-logging and rules of logarithms (Questions 7(ii) and 7(iv))
- Failure to give clear descriptions in the comprehension paper (Questions 4 and 7)
- Inappropriate accuracy, for example in Question 2, candidates either gave insufficient accuracy (answers to the nearest integer) or they gave too much accuracy (answers to 4 or more decimal places) – candidates are reminded to give answers to 1 decimal place for questions involving trigonometry
- Failure to give exact answers when required (Question 5(iii))
- Failure to give sufficient detail when verifying given results (Questions 5(i), 5(ii), 6(i), 6(ii)(A), 7(ii), 7(iv))

Some candidates still assume that showing that a vector is perpendicular to one vector in the plane is sufficient to show that it is a normal vector.

Quite a number of candidates failed to attempt some parts but there did not appear to be a shortage of time for either Paper.

Centres are again reminded that as Papers A and B are marked separately any supplementary sheets used must be attached to the appropriate paper.

Comments on Individual Questions:

Paper A

Question 1

Common errors included:

- $5(x+1)-3(2x+1)=(2x+1)(x+1)$ (and not the correct $5x(x+1)-...$)
- Expanding $-3(2x+1)$ as either $-6x+3$ or $-6x-1$ or $-6x+1$
- There were some candidates who did not multiply up on the right-hand side and so obtained $5x(x+1)-3(2x+1)=1$
- Some lost the final two marks for not applying the quadratic formula correctly

However, this question was generally done well with most candidates scoring full marks and demonstrating sound basic algebraic manipulation skills. It was common to see the use of the quadratic formula as much as factorising to solve the final quadratic equation. Very few completed the square but those that did were mainly successful.

Question 2

The majority of candidates correctly replaced $\cos 2\theta$ with $1-2\sin^2\theta$ although a minority of candidates made the costly mistake of replacing $\cos 2\theta$ with $1-\sin^2\theta$. While some candidates struggled to factorise $12\sin^2\theta - \sin\theta - 6 = 0$ many used the quadratic formula to solve this equation, and as with question 1, there were some candidates who did not state or apply the quadratic formula correctly. While the majority of candidates found the correct values for $\sin\theta$ some incorrectly obtained $-\dfrac{3}{4}$ and/or $\dfrac{2}{3}$. Of those candidates that obtained the correct values for $\sin\theta$ the majority went on to score full marks. However, it was fairly common to see '

$\arcsin\left(-\dfrac{2}{3}\right) = -41.81...$ *therefore no solutions in the range'* with no appreciation that solutions in

the correct range could be found from this value. Having found the principal values it was common for candidates to get the other solutions in the range, often sketching the sine curve to help them, though most did this correctly without demonstrating any method.

Question 3

The most common mistake in part (i) was to use a value of 2 rather than -2 as the coefficient of x in each term of the expansion. The binomial coefficients were nearly always correct though a small number missed the $2!$ from the denominator of the x^2 term. While the majority of candidates used the correct value of n a small minority incorrectly used $n = \dfrac{1}{3}$ or $-\dfrac{2}{3}$. The range of validity of the expansion was done much better than in previous years although the most common mistake was to give non-strict inequalities. Other mistakes included:

- $-\dfrac{1}{2} < -x < \dfrac{1}{2}$

- $|x| > \dfrac{1}{2}$

- $\dfrac{1}{2} < x < -\dfrac{1}{2}$

In part (ii) the majority of candidates correctly multiplied their answer from part (i) with $(1-3x)$ and simplified this expression correctly to obtain the correct values of a and b. It was concerning, however, that a number of candidates wrote

$(1-3x)(1+\dfrac{2}{3}x+\dfrac{8}{9}x^2+...) = 1+\dfrac{2}{3}x+\dfrac{8}{9}x^2-3x-2x-\dfrac{8}{3}x^2+...$ or even more worryingly expanded

$(1-3x)^1$ as $1-3x+$ higher order terms in x.

Question 4

This question differentiated well due to the coefficient of $\sin x$ taking the form of a positive constant rather than a number. Many candidates, however, were unfazed by this and worked out the correct values for R and α. Some candidates lost the first method mark by not including R in the expanded trigonometric statements $R\cos\alpha = 1, R\sin\alpha = \lambda$. Writing α in terms of the more complex $\arcsin$ and $\arccos$ expressions was surprisingly common.

It was a little worrying that a sizeable minority of candidates went from the correct $R = \sqrt{1+\lambda^2}$ to the incorrect $R = 1+\lambda$, thinking the squared terms and the square root cancelled each other out.

In part (ii) those candidates that realised that $R = 2$ usually went on to get the correct values for λ and α. However it was common for λ to be incorrect due to an incorrect expression for R from part (i). A fair proportion of candidates gave α in degrees and those who gave α as either

$$\arccos\left(\frac{1}{\sqrt{1+\lambda^2}}\right) \text{ or } \arcsin\left(\frac{\lambda}{\sqrt{1+\lambda^2}}\right)$$ were generally less successful in this part than those who

gave α as $\arctan\lambda$.

Question 5

In part (i) the majority of candidates correctly wrote down the result that $\dfrac{dy}{dx} = \dfrac{2\sec^2\theta}{\sec\theta\tan\theta}$. It was

clear though that a significant number of candidates did not know that $\dfrac{d}{d\theta}(\tan\theta) = \sec^2\theta$, and the

majority of these applied the quotient rule to $\dfrac{\sin\theta}{\cos\theta}$ in an attempt to derive this result. A significant

minority of candidates did not show adequate working to gain the final mark in part (i) with many

going directly from $\dfrac{2\sec\theta}{\tan\theta}$ (or equivalent) to the given answer. Candidates are reminded that in

questions in which the demand is to show a given result then sufficient working must be shown to satisfy the examiner that the answer has been correctly derived and not simply written down. It was

also noted by examiners that a number of candidates multiplied their expressions for $\dfrac{dy}{d\theta}$ and $\dfrac{dx}{d\theta}$

rather than dividing.

Although the techniques of verifying the Cartesian equation of the curve was relatively straightforward, answers in part (ii) were poorly presented, with many candidates unable to present a mathematical justification clearly and formally. Some candidates were insufficiently confident in the use of standard trigonometric identities and for some candidates it was as if they had never seen the relationship that $1 + \tan^2\theta \equiv \sec^2\theta$.

In part (iii) the vast majority of candidates considered both the correct integral for the volume of revolution and integrated $4x^2 - 4$ correctly. It was the mention of a rotation of $180°$ that seemed to

concern many candidates and a considerable number divided the correct answer of $\dfrac{16}{3}\pi$ by 2.

Question 6

There seemed to be a misconception amongst a number of candidates about the definition of 'show' and 'verify' in this first part. The majority of candidates correctly substituted the coordinates of A, B and E into the equation of the plane, to achieve the first two marks. Those candidates who tried to derive the equation of the plane had methods that were generally incomplete and despite a lot of work, did not gain much credit.

In part (ii)(A) a number of candidates seemed to think that just showing one direction vector was normal to the plane was sufficient and some candidates showed all three. It was unfortunate that so many lost marks by not showing the evaluation of the scalar product(s) even though this was another 'show that' question and so examiners had to be convinced that the candidates were indeed showing the required results and not simply stating them.

Part (ii)(B) was nearly always done correctly with only a small minority having the equation of the plane as $x - 6y - 6 = 0$ rather than the correct $x - 6y + 6 = 0$.

Part (ii)(C) occasionally contained simple arithmetical errors with the correct equation often leading to either $b = \dfrac{3}{4}$ or $b = -\dfrac{4}{3}$. Some candidates stated the direction vector $\overrightarrow{FG}$ correctly and then either did not state the length of FG or resorted to using Pythagoras to calculate this length, not recognising that they could simply read off the required answer.

To find the angle EFB in part (iii) most candidates attempted to find vectors (and hence use the scalar product) that would lead to the correct angle, though there were common arithmetical errors here.

Some candidates used $\overrightarrow{EF} \cdot \overrightarrow{FB}$ and hence obtained an acute angle, often not recognising that they needed to subtract this from $180°$ as angle EFB was obtuse. Some candidates who used $\overrightarrow{FE}$ and $\overrightarrow{FB}$ 'lost' the minus sign in their calculation and ended up with an answer of $36.9°$. A very small number of candidates calculated the lengths of the three sides of triangle EFB and used the cosine rule.

Part (iv) required candidates to find the height of Q above the ground. Those candidates who thought the $y-$ axis represented the vertical height clearly did not look at the diagram carefully enough. A surprising number of candidates could not calculate the z coordinate of P correctly. Some candidates seemed to think that in finding $\overrightarrow{PQ}$ they had arrived at the required answer whereas other candidates worked out the direct distance from the origin to point Q. Too many candidates worked out both the x and y coordinates of Q when only the z coordinate was required. Finally, a number of candidates found the correct position vector for Q but did not explicitly state the height of Q above the ground.

Question 7

Part (i) was answered extremely well with nearly all candidates correctly expressing $\dfrac{1}{(1+2x)(1-x)}$ in partial fractions.

In part (ii) the majority of candidates were able to separate the variables and substitute their partial fractions correctly. There were, however, frequent errors in the integration usually when candidates forgot to divide by 2 when integrating $\dfrac{1}{1+2x}$ and/or when they forgot to do the same process with the -1 when integrating $\dfrac{1}{1-x}$. Many candidates did not include a constant of integration or, if included, it was either subsequently ignored or set to zero without any mathematical justification. Most candidates were able to combine their logarithmic terms correctly, though examiners noted the high volume of cases in which the 'correct' printed answer was seen following earlier incorrect working.

Most candidates achieved the first two marks in part (iii) for finding the value of k, but many made errors in handling the logarithms to find the time taken for the drug concentration to reach 90% of its maximum value. Examiners noted the large variation in the accuracy of candidates' final answers in this part.

In part (iv) most candidates multiplied up by $1-x$, collected and factorised the x terms correctly. The main problem seemed to be how to get the negative exponent. These often appeared when candidates divided $e^{3kt} - 1$ by $2 + e^{3kt}$, losing the final accuracy mark in the process. It was also common for candidates to simply not show how $\dfrac{e^{3kt} - 1}{2 + e^{3kt}}$ was equal to the given answer. Those candidates who started by taking the reciprocal of the answer given in part (ii) were usually far

more successful in deriving the required result in this part. The majority of candidates who attempted to verify that the drug concentration approached its maximum value in the long term recognised that as $t \to \infty, e^{-3kt} \to 0$, although some candidates simply substituted a large value of t to show that x was close to 1, this approach was not sufficient to earn the final mark in this part.

Paper B

Question 1

Most candidates scored all three marks.

Question 2

While the vast majority gave the correct answer as a percentage a number of candidates left their answer as a decimal. The most common errors included numerators that were not equal to 110 (the most common incorrect values being either 90 or 95 or 125) and/or denominators that were not equal to 146.

Question 3

This part was nearly always answered correctly although the question did specifically asked for an answer to 2 decimal places and a significant number of candidates gave an answer of only 5.2. A smaller minority forgot the factor of 10 giving an answer of 0.52.

Question 4

Candidates found this part relatively demanding with very few fully explaining how the figure of 188 seconds had been derived. While many candidates gave a correct calculation (and so scored the first mark) many did not give an accurate and correct justification for their calculation. Many candidates who gave the departure time of 164 seconds did not make it explicitly clear that they were talking about the first 8 floors of the building.

Question 5

Most candidates scored all three marks in part (i) with only occasional errors with the arrival time of the lift at the ground floor. Part (ii) differentiated well with only a minority of candidates obtaining the correct answer of 392 seconds.

Question 6

This part was nearly always correct although a significant number of candidates tried to apply their knowledge of probability distributions instead of using the more expected route of applying the given formula of $1 - \left(\dfrac{f-1}{f} \right)^n$.

Question 7

The majority of candidates struggled with this final part and it was clear that many did not understand how to tackle this question. It was expected that the candidates would simply use Table 2 with a value of $n = 15$ and read off the two values of f at 6 and 4 and obtain 5.6 and 3.9 respectively. They could then have concluded that these two values suggest that it is indeed a common occurrence for the cars to stop on every floor that they serve. Many candidates, however, gave an answer that was not mathematical or numerical in nature and so therefore scored no marks in this final part.

GCE Mathematics (MEI)

				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	48	43	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	56	50	44	39	34	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	56	51	46	41	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	74	67	60	54	48	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	62	57	53	49	45	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	59	52	46	40	34	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	38	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	62	54	46	39	32	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	54	47	40	33	27	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	64	56	48	41	34	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	65	60	55	50	46	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	64	58	52	47	42	0
			UMS	100	80	70	60	50	40	u
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	56	51	46	41	37	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	54	49	44	39	34	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	45	40	34	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
			UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	54	47	41	35	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	54	47	41	35	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	56	51	46	41	37	0
		UMS	100	80	70	60	50	40	0

OCR

Oxford Cambridge and RSA

Tuesday 16 June 2015 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01B Applications of Advanced Mathematics (C4) Paper B: Comprehension

QUESTION PAPER

Duration: Up to 1 hour

Candidate forename		Candidate surname	

Centre number					Candidate number				
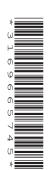
INSTRUCTIONS TO CANDIDATES

* The Insert will be found inside this document.
* Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
* Use black ink. HB pencil may be used for graphs and diagrams only.
* Answer **all** the questions.
* Read each question carefully. Make sure you know what you have to do before starting your answer.
* Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
* Do **not** write in the bar codes.
* The Insert contains the text for use with the questions.
* You are permitted to use a scientific or graphical calculator in this paper.
* Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

* The number of marks is given in brackets **[ ]** at the end of each question or part question.
* You may find it helpful to make notes and do some calculations as you read the passage.
* You are **not** required to hand in these notes with your question paper.
* You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
* The total number of marks for this paper is **18**.
* This document consists of **8** pages. Any blank pages are indicated.

OCR is an exempt Charity

Turn over

1 In a building with a single lift, the car is located on the ground floor. The following calls arrive in quick succession, all before the car has started on its upward journey.

➢ Passenger P on floor 9 calls for a down-car (intending to go to floor 1);
➢ Passenger Q on floor 5 calls for an up-car (intending to go to floor 10);
➢ Passenger R on floor 4 calls for a down-car (intending to go to floor 1).

The following events need to take place, but not necessarily in the order given.

E1. Pick up P
E2. Pick up Q
E3. Pick up R
E4. Drop off P and R
E5. Drop off Q

Assuming the car serves requests as described in lines 16 to 20,

  (i) which event occurs first? **[1]**

 (ii) which event occurs second? **[1]**

(iii) which event occurs last? **[1]**

1 (i)	
1 (ii)	
1 (iii)	

PLEASE DO NOT WRITE IN THIS SPACE

2 In line 79 it says "For most journeys, more than half the journey time is composed of load time and transfer time". For what percentage of the journey time for the round trip made by car A in Table 4 is the car stationary? **[2]**

2	

3 Using the expression on line 51, work out the answer to the question on lines 39 and 40 for the case where there are 10 upper floors and 7 people. Give your answer to 2 decimal places. **[2]**

3	

4 In lines 89 and 90 it says "… on average there will be approximately 8 stops per trip. A round trip with 8 stops would take between 188 and 200 seconds". Explain how the figure of 188 seconds has been derived. **[2]**

4	

5 **(i)** Referring to Strategy 3 and lines 99 to 101, complete the table below for car C. **[3]**

(ii) Calculate the time car C will take to transport all the people who work on floors 7 and 8, and return to the ground floor. **[1]**

5(i)			

	Car C	
	Arrival time (seconds)	Departure time (seconds)
Ground floor	0	20
Floor 1		
Floor 2		
Floor 3		
Floor 4		
Floor 5		
Floor 6		
Floor 7		
Floor 8		
Floor 9		
Floor 10		
Return to ground floor		

5(ii)

6 8 people make independent visits to any one of the upper floors of a building with 10 upper floors. What is the probability that at least one of the visitors goes to the top floor? **[2]**

6

7 On lines 94 and 95 it says "Table 4 gives the timings for round trips in which the cars are required to stop at every floor they serve; Table 2 suggests this is a common occurrence in this case". Explain how Table 2 is used to make this claim. **[3]**

7

END OF QUESTION PAPER

6

BLANK PAGE

PLEASE DO NOT WRITE ON THIS PAGE

BLANK PAGE

PLEASE DO NOT WRITE ON THIS PAGE

PLEASE DO NOT WRITE ON THIS PAGE

OCR
Oxford Cambridge and RSA

Tuesday 16 June 2015 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01B Applications of Advanced Mathematics (C4) Paper B: Comprehension

INSERT

Duration: Up to 1 hour

INFORMATION FOR CANDIDATES

* This Insert contains the text for use with the questions.
* This document consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

* Do not send this Insert for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Scheduling lifts

Introduction

Most modern high-rise office buildings have several lift shafts located together around a central point in the building. Fig. 1 illustrates the plan view of a possible layout for each floor of a building having four lifts arranged around a hall area.

Lift Shaft A	Lift Shaft B

Hall area

Lift Shaft C	Lift Shaft D

Fig. 1

In each lift shaft, a car travels up and down between the floors of the building.

When calling a car from the hall area, the passenger presses either an 'up' button or a 'down' button to indicate the direction in which he or she wishes to travel. The way in which the lifts are programmed will determine which lift stops for the passenger. The time that elapses between calling a car and the car arriving is called the *wait time*. On entering the car the passenger selects the floor number to which he or she wishes to travel.

To operate effectively, high-rise buildings depend on the efficiency of their lifts. Some buildings have cars that can stop at all floors; in other buildings some of the cars are restricted to certain floors. This article looks at several ways in which lifts are programmed in office buildings.

Buildings with a single lift

In a building served by just one lift, a typical upward then downward journey will be as follows. When moving upwards, the car will only stop to pick up passengers if they wish to travel upwards. When all these passengers have reached their destinations, the car will travel to the highest floor from which a downward request has been made. It will then travel downwards, satisfying other downward requests as it does so.

In some buildings, when there are no further requests the car is programmed to return to the ground floor since that is where most calls are made; this should be expected to minimise the average wait time for passengers.

Other methods can be employed in buildings with just one lift shaft to improve the efficiency of the lift. An example of such a method is that a lift could be programmed to stop at the even-numbered floors only. In such a building, a passenger who wants to reach the seventh floor would need to leave at the sixth or eighth floor. If this decision is taken during the construction of the building it has the financial benefit of not requiring investment in entrances on the odd-numbered floors. In addition it reduces *transfer time*; this is the time spent opening doors, transferring people in and out of the car and closing the doors. One

obvious drawback of this arrangement is the problem it causes for people who cannot negotiate stairs, such as those who use wheelchairs.

Average number of stops

Imagine that 7 people enter a car on the ground floor of a building which has 10 floors above the ground floor; these will be called *upper floors*. Assuming that the requests are independent and all floors are equally likely to be selected, the probability that these 7 people will all request different floors is

$$\frac{10}{10} \times \frac{9}{10} \times \frac{8}{10} \times \frac{7}{10} \times \frac{6}{10} \times \frac{5}{10} \times \frac{4}{10}.$$

This is approximately equal to 0.06. So the probability that there will be at least one floor where more than one person leaves the car is approximately 0.94. Thus it is very likely that the car will actually be required to stop at fewer than 7 floors. This raises the question: what is the average number of floors at which the car will stop?

To answer this question we assume that the requests are independent, that all floors are equally likely to be selected and that the car will not be making any stops to pick up additional passengers; this final assumption is realistic at the start of a working day. Letting n represent the number of passengers and f the number of upper floors, under the given assumptions the probability that nobody selects a particular floor is

$$\left(\frac{f-1}{f}\right)^{n}.$$

Therefore the probability that at least one person will select a particular floor is

$$1-\left(\frac{f-1}{f}\right)^{n}.$$

This probability is the same for all f floors. It gives the average proportion of floors at which the car stops. It follows that the average number of floors at which the car stops is

$$f\left\{1-\left(\frac{f-1}{f}\right)^{n}\right\}.$$

Table 2 shows the average numbers of floors at which cars carrying up to 15 passengers would stop in buildings with up to 10 upper floors; each value is given to an accuracy of one decimal place.

Number of passengers, n

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of upper floors, f	1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	2	1.0	1.5	1.8	1.9	1.9	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
	3	1.0	1.7	2.1	2.4	2.6	2.7	2.8	2.9	2.9	2.9	3.0	3.0	3.0	3.0	3.0
	4	1.0	1.8	2.3	2.7	3.1	3.3	3.5	3.6	3.7	3.8	3.8	3.9	3.9	3.9	3.9
	5	1.0	1.8	2.4	3.0	3.4	3.7	4.0	4.2	4.3	4.5	4.6	4.7	4.7	4.8	4.8
	6	1.0	1.8	2.5	3.1	3.6	4.0	4.3	4.6	4.8	5.0	5.2	5.3	5.4	5.5	5.6
	7	1.0	1.9	2.6	3.2	3.8	4.2	4.6	5.0	5.3	5.5	5.7	5.9	6.1	6.2	6.3
	8	1.0	1.9	2.6	3.3	3.9	4.4	4.9	5.3	5.6	5.9	6.2	6.4	6.6	6.8	6.9
	9	1.0	1.9	2.7	3.4	4.0	4.6	5.1	5.5	5.9	6.2	6.5	6.8	7.1	7.3	7.5
	10	1.0	1.9	2.7	3.4	4.1	4.7	5.2	5.7	6.1	6.5	6.9	7.2	7.5	7.7	7.9

Table 2

For example, in a building with 10 upper floors, a car carrying 15 passengers would, on average, stop at approximately 8 floors. $\qquad$ 55

Buildings with multiple lifts

It is not uncommon for more than 1000 people to work in a single high-rise office building. When designing these buildings, the architect has to reach a compromise between allocating space to lift shafts and space to offices. It is normal to find a hall area in such buildings that is occupied by several lift shafts. These lifts are programmed to operate in different ways depending on the expected demands at different 60 times of day.

Upward peak-time operation

During the morning rush hour, when people arrive for work, or at the end of the lunch break, the cars are programmed to return to the ground floor. Each car leaves when it reaches its maximum passenger load or it has had its doors open for a certain period of time. $\qquad$ 65

For the rest of this article we will consider an office building with a ground floor and 10 upper floors served by 4 lifts, A, B, C and D, which are grouped together around a hall area. Each car can hold 15 people. 30 people work on each of the upper floors.

To model the use of the lifts at upward peak-time, we make the following simplifying assumptions.

- On the ground floor there are always 15 people waiting to enter any given car. 70
- For any car on the ground floor, the total time taken for its doors to open, 15 people to enter and the doors to close is 20 seconds; this is called the *loading time*.
- Each car takes 3 seconds to move between adjacent floors.
- On an upper floor the transfer time, to open the car doors, let passengers out and close the doors, is 15 seconds. (It is assumed nobody enters a car on an upper floor during this peak 75 time.)
- Nobody uses the stairs to walk between floors.

There are many ways in which the lifts could be programmed to transfer these 300 people to their offices. For most journeys, more than half the journey time is composed of load time and transfer time; minimising this is a major consideration when designing an effective strategy. Three different strategies 80 are considered below.

Strategy 1

All four cars serve every floor. On the return journey from the top floor to the ground floor, nobody enters the car and so the descent takes 30 seconds. Table 3 gives the timings for a round trip for one car that is required to stop at every floor.

	Arrival time (seconds)	Departure time (seconds)
Ground floor	0	20
Floor 1	23	38
Floor 2	41	56
Floor 3	59	74
Floor 4	77	92
Floor 5	95	110
Floor 6	113	128
Floor 7	131	146
Floor 8	149	164
Floor 9	167	182
Floor 10	185	200
Return to ground floor	230	

Table 3

With this strategy, and assuming every car stops at every floor in each trip, it will require 5 trips for each of the 4 cars to transport the 300 people to their floors. It will take 19 minutes 10 seconds for the cars to complete these trips and return to the ground floor.

However, from Table 2 it can be seen that on average there will be approximately 8 stops per trip. A round trip with 8 stops would take between 188 and 200 seconds.

Strategy 2

All 4 cars serve the ground floor. In addition, cars A and B serve floors 1 to 6 and cars C and D serve floors 7 to 10.

Table 4 gives the timings for round trips in which the cars are required to stop at every floor they serve; Table 2 suggests this is a common occurrence in this case.

95

	Cars A and B		Cars C and D	
	Arrival time (seconds)	Departure time (seconds)	Arrival time (seconds)	Departure time (seconds)
Ground floor	0	20	0	20
Floor 1	23	38		
Floor 2	41	56		
Floor 3	59	74		
Floor 4	77	92		
Floor 5	95	110		
Floor 6	113	128		
Floor 7			41	56
Floor 8			59	74
Floor 9			77	92
Floor 10			95	110
Return to ground floor	146		140	

Table 4

With this strategy, and assuming every car stops at every floor it serves in each trip, it will take 14 minutes 36 seconds for the 4 cars to transport the 300 people to their floors and return to the ground floor.

Strategy 3

All 4 cars serve the ground floor. In addition, car A serves floors 1 to 3, car B serves floors 4 to 6, car C serves floors 7 and 8, and car D serves floors 9 and 10.

100

Assuming the cars are required to stop at every floor they serve, with this strategy car B is going to take longer than car A to complete each of its trips, and car D is going to take longer than car C.

One round trip for each of cars B and D is summarised in Table 5.

	Car B		Car D	
	Arrival time (seconds)	Departure time (seconds)	Arrival time (seconds)	Departure time (seconds)
Ground floor	0	20	0	20
Floor 1				
Floor 2				
Floor 3				
Floor 4	32	47		
Floor 5	50	65		
Floor 6	68	83		
Floor 7				
Floor 8				
Floor 9			47	62
Floor 10			65	80
Return to ground floor	101		110	

Table 5

Car B will be the last to complete its trips, returning to the ground floor after 10 minutes 6 seconds. This would seem to be the most efficient strategy but the assumption that the cars will always be filled in 20 seconds at the ground floor despite the fact that they serve only 2 or 3 floors is, perhaps, not realistic.

Other strategies are possible. In taller buildings, express lifts are often used. For example, in a building with 50 floors, an express lift might take people from the ground floor to floor 40 without stopping, and another lift then serves all higher floors.

Downward peak-time operation

At the start of the lunch break or the end of the working day, a high percentage of the office workers will need to travel to the ground floor in a short period of time. At these times, cars can be programmed to occupy the highest floors, each car on a different floor, to await a call.

Off-peak operation

During the working day, the office workers move between floors and in and out of the building. Whereas at peak times the cars mainly carry passengers in one direction, at off-peak times this is not the case; round trips often involve similar numbers of people travelling up and travelling down. In order to minimise wait time for passengers, different strategies are needed and these depend on several factors including the distribution of the population and the relative attraction of each floor.

Conclusion

In this article several simplifying assumptions have been made. In reality, to improve the efficiency of the lifts, it is necessary to carry out an in-depth study of the ways in which the users of the building use the lifts. Complex mathematical models are created to simulate lift use at different times of day and these provide optimal strategies for the lifts so that buildings can function efficiently.

1. **Annotations and abbreviations**

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2.   **Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand**

     a   Annotations should be used whenever appropriate during your marking.

           The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

           For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

     b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

           Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

     c   The following types of marks are available.

           M
           A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

           A
           Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

           B
           Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g   Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(i)		E2. Pick up Q	B1 **[1]**	
1	(ii)		E5. Drop off Q	B1 **[1]**	
1	(iii)		E4. Drop off P and R	B1 **[1]**	
2			$\dfrac{110}{146} \times 100\% = 75.3\%$	M1 A1 **[2]**	M1 allow (90 or 95 or 125) / 146 $\times 100$ 75.3 or better (eg 75.34 etc), 75 is A0 B2 75.3 with no working, B1 0.753 (oe),
3			$10\left(1-\left(\dfrac{9}{10}\right)^7\right)=5.22$	M1 A1 **[2]**	Substitution of the correct values into the correct formula 5.22 (must be 2 dp), B2 5.22 www (isw after correct answer seen)
4			 Lift calls at floors 1 to 8 (only), leaving floor 8 at 164 seconds. Descent takes 24 seconds	B1* E1dep* **[2]**	Correct calculation eg 164 + 24 or 230 − (30 + 12) (oe) but not 200 − 12 Correct justification of the correct calculation (dependent on B1). If the departure time of 164 is used then their explanation must imply the use of the first 8 floors (not just '8 floors')

Question		Answer	Marks	Guidance
5	(i)	See table below	B1 B1 B1 **[3]**	 59, 74 **or** their 56 + 3, their 59 + 15 98 or their 74 + 24
5	(ii)	$4 \times 98s = 392s$ (6 mins 32s)	B1 **[1]**	cao
6		$1 - \left(\dfrac{9}{10} \right)^8 = 0.5695...$	M1 A1 **[2]**	Substitution of the correct values into the correct formula Accept 2 to 4 dp
7		$n = 15, f = 6$ gives 5.6	B1	5.6
		$n = 15, f = 4$ gives 3.9	B1	3.9
		so 'usually' 6 and 4 stops respectively	E1 **[3]**	Dependent on B1B1 – additional values of f considered E0

Table for Question 5 (i):

	Car C	
	Arrival time (seconds)	Departure time (seconds)
Ground floor	0	20
Floor 7	41	56
Floor 8	59	74
Return to ground floor	98	

4754 Applications of Advanced Mathematics (C4)

General Comments

Candidates found Paper A more straightforward than in previous years and therefore the standard of work was very high. This paper was accessible to all candidates but there were sufficient questions for the more able candidates to show their skills.

Paper B, the comprehension, was well understood and most candidates scored good marks here.

Candidates made similar errors as in previous years and these included:

- Sign and basic algebraic errors (Question 1)
- Failure to include a constant of integration (Question 7(ii))
- Poor anti-logging and rules of logarithms (Questions 7(ii) and 7(iv))
- Failure to give clear descriptions in the comprehension paper (Questions 4 and 7)
- Inappropriate accuracy, for example in Question 2, candidates either gave insufficient accuracy (answers to the nearest integer) or they gave too much accuracy (answers to 4 or more decimal places) – candidates are reminded to give answers to 1 decimal place for questions involving trigonometry
- Failure to give exact answers when required (Question 5(iii))
- Failure to give sufficient detail when verifying given results (Questions 5(i), 5(ii), 6(i), 6(ii)(A), 7(ii), 7(iv))

Some candidates still assume that showing that a vector is perpendicular to one vector in the plane is sufficient to show that it is a normal vector.

Quite a number of candidates failed to attempt some parts but there did not appear to be a shortage of time for either Paper.

Centres are again reminded that as Papers A and B are marked separately any supplementary sheets used must be attached to the appropriate paper.

Comments on Individual Questions:

Paper A

Question 1

Common errors included:

- $5(x+1)-3(2x+1)=(2x+1)(x+1)$ (and not the correct $5x(x+1)-...$)
- Expanding $-3(2x+1)$ as either $-6x+3$ or $-6x-1$ or $-6x+1$
- There were some candidates who did not multiply up on the right-hand side and so obtained $5x(x+1)-3(2x+1)=1$
- Some lost the final two marks for not applying the quadratic formula correctly

However, this question was generally done well with most candidates scoring full marks and demonstrating sound basic algebraic manipulation skills. It was common to see the use of the quadratic formula as much as factorising to solve the final quadratic equation. Very few completed the square but those that did were mainly successful.

Question 2

The majority of candidates correctly replaced $\cos 2\theta$ with $1 - 2\sin^2\theta$ although a minority of candidates made the costly mistake of replacing $\cos 2\theta$ with $1 - \sin^2\theta$. While some candidates struggled to factorise $12\sin^2\theta - \sin\theta - 6 = 0$ many used the quadratic formula to solve this equation, and as with question 1, there were some candidates who did not state or apply the quadratic formula correctly. While the majority of candidates found the correct values for $\sin\theta$ some incorrectly obtained $-\dfrac{3}{4}$ and/or $\dfrac{2}{3}$. Of those candidates that obtained the correct values for $\sin\theta$ the majority went on to score full marks. However, it was fairly common to see '

$\arcsin\left(-\dfrac{2}{3}\right) = -41.81...$ *therefore no solutions in the range'* with no appreciation that solutions in the correct range could be found from this value. Having found the principal values it was common for candidates to get the other solutions in the range, often sketching the sine curve to help them, though most did this correctly without demonstrating any method.

Question 3

The most common mistake in part (i) was to use a value of 2 rather than -2 as the coefficient of x in each term of the expansion. The binomial coefficients were nearly always correct though a small number missed the $2!$ from the denominator of the x^2 term. While the majority of candidates used the correct value of n a small minority incorrectly used $n = \dfrac{1}{3}$ or $-\dfrac{2}{3}$. The range of validity of the expansion was done much better than in previous years although the most common mistake was to give non-strict inequalities. Other mistakes included:

- $-\dfrac{1}{2} < -x < \dfrac{1}{2}$

- $|x| > \dfrac{1}{2}$

- $\dfrac{1}{2} < x < -\dfrac{1}{2}$

In part (ii) the majority of candidates correctly multiplied their answer from part (i) with $(1 - 3x)$ and simplified this expression correctly to obtain the correct values of a and b. It was concerning, however, that a number of candidates wrote

$(1 - 3x)(1 + \dfrac{2}{3}x + \dfrac{8}{9}x^2 + ...) = 1 + \dfrac{2}{3}x + \dfrac{8}{9}x^2 - 3x - 2x - \dfrac{8}{3}x^2 + ...$ or even more worryingly expanded

$(1 - 3x)^1$ as $1 - 3x +$ higher order terms in x.

Question 4

This question differentiated well due to the coefficient of $\sin x$ taking the form of a positive constant rather than a number. Many candidates, however, were unfazed by this and worked out the correct values for R and α. Some candidates lost the first method mark by not including R in the expanded trigonometric statements $R\cos\alpha = 1, R\sin\alpha = \lambda$. Writing α in terms of the more complex $\arcsin$ and $\arccos$ expressions was surprisingly common.

It was a little worrying that a sizeable minority of candidates went from the correct $R = \sqrt{1 + \lambda^2}$ to the incorrect $R = 1 + \lambda$, thinking the squared terms and the square root cancelled each other out.

In part (ii) those candidates that realised that $R = 2$ usually went on to get the correct values for λ and α. However it was common for λ to be incorrect due to an incorrect expression for R from part (i). A fair proportion of candidates gave α in degrees and those who gave α as either

$$\arccos\left(\frac{1}{\sqrt{1+\lambda^2}}\right) \text{ or } \arcsin\left(\frac{\lambda}{\sqrt{1+\lambda^2}}\right)$$ were generally less successful in this part than those who

gave α as $\arctan \lambda$.

Question 5

In part (i) the majority of candidates correctly wrote down the result that $\dfrac{dy}{dx} = \dfrac{2\sec^2\theta}{\sec\theta\tan\theta}$. It was

clear though that a significant number of candidates did not know that $\dfrac{d}{d\theta}(\tan\theta) = \sec^2\theta$, and the

majority of these applied the quotient rule to $\dfrac{\sin\theta}{\cos\theta}$ in an attempt to derive this result. A significant

minority of candidates did not show adequate working to gain the final mark in part (i) with many

going directly from $\dfrac{2\sec\theta}{\tan\theta}$ (or equivalent) to the given answer. Candidates are reminded that in

questions in which the demand is to show a given result then sufficient working must be shown to satisfy the examiner that the answer has been correctly derived and not simply written down. It was

also noted by examiners that a number of candidates multiplied their expressions for $\dfrac{dy}{d\theta}$ and $\dfrac{dx}{d\theta}$

rather than dividing.

Although the techniques of verifying the Cartesian equation of the curve was relatively straightforward, answers in part (ii) were poorly presented, with many candidates unable to present a mathematical justification clearly and formally. Some candidates were insufficiently confident in the use of standard trigonometric identities and for some candidates it was as if they had never seen the relationship that $1 + \tan^2\theta \equiv \sec^2\theta$.

In part (iii) the vast majority of candidates considered both the correct integral for the volume of revolution and integrated $4x^2 - 4$ correctly. It was the mention of a rotation of $180°$ that seemed to

concern many candidates and a considerable number divided the correct answer of $\dfrac{16}{3}\pi$ by 2.

Question 6

There seemed to be a misconception amongst a number of candidates about the definition of 'show' and 'verify' in this first part. The majority of candidates correctly substituted the coordinates of A, B and E into the equation of the plane, to achieve the first two marks. Those candidates who tried to derive the equation of the plane had methods that were generally incomplete and despite a lot of work, did not gain much credit.

In part (ii)(A) a number of candidates seemed to think that just showing one direction vector was normal to the plane was sufficient and some candidates showed all three. It was unfortunate that so many lost marks by not showing the evaluation of the scalar product(s) even though this was another 'show that' question and so examiners had to be convinced that the candidates were indeed showing the required results and not simply stating them.

Part (ii)(B) was nearly always done correctly with only a small minority having the equation of the plane as $x - 6y - 6 = 0$ rather than the correct $x - 6y + 6 = 0$.

Part (ii)(C) occasionally contained simple arithmetical errors with the correct equation often leading to either $b = \dfrac{3}{4}$ or $b = -\dfrac{4}{3}$. Some candidates stated the direction vector $\overrightarrow{FG}$ correctly and then either did not state the length of FG or resorted to using Pythagoras to calculate this length, not recognising that they could simply read off the required answer.

To find the angle EFB in part (iii) most candidates attempted to find vectors (and hence use the scalar product) that would lead to the correct angle, though there were common arithmetical errors here.

Some candidates used $\overrightarrow{EF} \cdot \overrightarrow{FB}$ and hence obtained an acute angle, often not recognising that they needed to subtract this from $180°$ as angle EFB was obtuse. Some candidates who used $\overrightarrow{FE}$ and $\overrightarrow{FB}$ 'lost' the minus sign in their calculation and ended up with an answer of $36.9°$. A very small number of candidates calculated the lengths of the three sides of triangle EFB and used the cosine rule.

Part (iv) required candidates to find the height of Q above the ground. Those candidates who thought the $y-$ axis represented the vertical height clearly did not look at the diagram carefully enough. A surprising number of candidates could not calculate the z coordinate of P correctly. Some candidates seemed to think that in finding $\overrightarrow{PQ}$ they had arrived at the required answer whereas other candidates worked out the direct distance from the origin to point Q. Too many candidates worked out both the x and y coordinates of Q when only the z coordinate was required. Finally, a number of candidates found the correct position vector for Q but did not explicitly state the height of Q above the ground.

Question 7

Part (i) was answered extremely well with nearly all candidates correctly expressing $\dfrac{1}{(1+2x)(1-x)}$ in partial fractions.

In part (ii) the majority of candidates were able to separate the variables and substitute their partial fractions correctly. There were, however, frequent errors in the integration usually when candidates forgot to divide by 2 when integrating $\dfrac{1}{1+2x}$ and/or when they forgot to do the same process with the -1 when integrating $\dfrac{1}{1-x}$. Many candidates did not include a constant of integration or, if included, it was either subsequently ignored or set to zero without any mathematical justification. Most candidates were able to combine their logarithmic terms correctly, though examiners noted the high volume of cases in which the 'correct' printed answer was seen following earlier incorrect working.

Most candidates achieved the first two marks in part (iii) for finding the value of k, but many made errors in handling the logarithms to find the time taken for the drug concentration to reach 90% of its maximum value. Examiners noted the large variation in the accuracy of candidates' final answers in this part.

In part (iv) most candidates multiplied up by $1-x$, collected and factorised the x terms correctly. The main problem seemed to be how to get the negative exponent. These often appeared when candidates divided $e^{3kt} - 1$ by $2 + e^{3kt}$, losing the final accuracy mark in the process. It was also common for candidates to simply not show how $\dfrac{e^{3kt}-1}{2+e^{3kt}}$ was equal to the given answer. Those candidates who started by taking the reciprocal of the answer given in part (ii) were usually far

more successful in deriving the required result in this part. The majority of candidates who attempted to verify that the drug concentration approached its maximum value in the long term recognised that as $t \to \infty, \mathrm{e}^{-3kt} \to 0$, although some candidates simply substituted a large value of t to show that x was close to 1, this approach was not sufficient to earn the final mark in this part.

Paper B

Question 1

Most candidates scored all three marks.

Question 2

While the vast majority gave the correct answer as a percentage a number of candidates left their answer as a decimal. The most common errors included numerators that were not equal to 110 (the most common incorrect values being either 90 or 95 or 125) and/or denominators that were not equal to 146.

Question 3

This part was nearly always answered correctly although the question did specifically asked for an answer to 2 decimal places and a significant number of candidates gave an answer of only 5.2. A smaller minority forgot the factor of 10 giving an answer of 0.52.

Question 4

Candidates found this part relatively demanding with very few fully explaining how the figure of 188 seconds had been derived. While many candidates gave a correct calculation (and so scored the first mark) many did not give an accurate and correct justification for their calculation. Many candidates who gave the departure time of 164 seconds did not make it explicitly clear that they were talking about the first 8 floors of the building.

Question 5

Most candidates scored all three marks in part (i) with only occasional errors with the arrival time of the lift at the ground floor. Part (ii) differentiated well with only a minority of candidates obtaining the correct answer of 392 seconds.

Question 6

This part was nearly always correct although a significant number of candidates tried to apply their knowledge of probability distributions instead of using the more expected route of applying the given formula of $1 - \left(\dfrac{f-1}{f} \right)^n$.

Question 7

The majority of candidates struggled with this final part and it was clear that many did not understand how to tackle this question. It was expected that the candidates would simply use Table 2 with a value of $n = 15$ and read off the two values of f at 6 and 4 and obtain 5.6 and 3.9 respectively. They could then have concluded that these two values suggest that it is indeed a common occurrence for the cars to stop on every floor that they serve. Many candidates, however, gave an answer that was not mathematical or numerical in nature and so therefore scored no marks in this final part.

GCE Mathematics (MEI)

				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	48	43	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	56	50	44	39	34	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	56	51	46	41	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	74	67	60	54	48	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	62	57	53	49	45	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	59	52	46	40	34	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	38	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	62	54	46	39	32	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	54	47	40	33	27	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	64	56	48	41	34	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	65	60	55	50	46	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	64	58	52	47	42	0
			UMS	100	80	70	60	50	40	u
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	56	51	46	41	37	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	54	49	44	39	34	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	45	40	34	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
			UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	54	47	41	35	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	54	47	41	35	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	56	51	46	41	37	0
		UMS	100	80	70	60	50	40	0